

# $R^4$ Couplings in $M$ and Type $II$ Theories on Calabi-Yau spaces

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We discuss several implications of  $R^4$  couplings in  $M$  theory when compactified on Calabi-Yau (CY) manifolds. In particular, these couplings can be predicted by supersymmetry from the mixed gauge-gravitational Chern-Simons couplings in five dimensions and are related to the one-loop holomorphic anomaly in four-dimensional  $N = 2$  theories. We find a new contribution to the Einstein term in five dimensions proportional to the Euler number of the internal CY threefold, which corresponds to a one-loop correction of the hypermultiplet geometry. This correction is reproduced by a direct computation in type  $II$  string theories. Finally, we discuss a universal non-perturbative correction to the type  $IIB$  hyper-metric.

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## 1. Introduction

It has recently been shown that certain  $R^4$  terms, present in type *IIA* and type *IIB* [1] theories, suggest the existence of similar terms in *M* theory [2]. In [2] a possible relation to the anomaly-cancelling  $A_3 \wedge I_8(R)$  term [3] due to supersymmetry was argued. In this note, we present further evidence of these terms looking at compactifications of *M* theory to lower odd-dimensional theories on Calabi-Yau manifolds. These compactifications are known to relate *M* theory to several dual partners, the most explored examples of the dualities being nine-dimensional duality between *M* theory on  $T^2$  and type *IIB* on  $S^1$  [4] and between *M* theory on  $X_n$  and heterotic string on  $X_{n-1} \times S^1$  in dimensions  $11 - 2n$ , where  $X_n$  is a CY manifold of complex dimension  $n$  (for recent reviews, see [5]).

In  $D = 9$  a one-loop calculation of the anomaly-generating function [6] gives a result similar to the one of the ten-dimensional type *IIA* calculation [7] and is of the form  $\int A_1 \wedge I_8(R)$ , where  $I_8(R)$  is an eight-form polynomial in curvature [3], and the vector can be identified with  $B_{\mu 9}$  in type *IIA* and with the Kaluza-Klein gauge boson  $g_{\mu 9}$  in type *IIB*. While in *M* theory this term is present already in eleven dimensions, ten-dimensional type *IIB* theory forbids a similar term [7]. However, its presence is supported by the  $T$ -duality between these two theories and is consistent with  $SL(2, Z)$  duality in nine dimensions. This will be discussed in detail in Section 4, where we also present a complementary argument and show the emergence of a chiral fivebrane in the nine-dimensional type *IIB* spectrum.

In  $D = 7$ ,  $R^4$  coupling on  $K3$  yields an  $R^2$  coupling needed to reproduce the dilaton equation of motion of heterotic string on  $T3$

$$\square \phi_7 = R^2 + \dots, \quad (1.1)$$

that is the supersymmetric extension of the string Bianchi identity. The heterotic dilaton  $\phi_7$  is given by the  $K3$  volume,  $e^{-2\phi_7} = V_{K3}$  [7,3,8].

The  $D = 5$  case will be discussed in detail in Section 2. We will show that the  $R^4$  terms generate two terms. One is the superpartner of the gravitational Chern-Simons term and is of the form

$$\sum_{h(1,1)} \alpha_\Lambda \int t^\Lambda R \wedge R \wedge e, \quad (1.2)$$

where  $t^\Lambda$  are five-dimensional special coordinates (subject to constraint  $\frac{1}{6} t^\Lambda t^\Delta t^\Sigma C_{\Lambda\Delta\Sigma} = 1$ ), and the constants  $\alpha_\Lambda$  will be defined in the next section. By further compactification to four dimensions, the two combine into

$$\sum_{h(1,1)} \alpha_\Lambda \int Z^\Lambda R^2, \quad (1.3)$$

where  $Z^\Lambda = A_5^\Lambda + it^\Lambda r_5$ ,  $r_5$  being the radius of the fifth dimension and  $A_\mu^\Lambda$  the five-dimensional gauge fields. The other remnant of  $R^4$  in  $D = 5$  is in the two-derivative part of the effective field theory; it is proportional to the Euler number and can be regarded as a correction to the hypermatter geometry. We reproduce this correction by a direct one-loop string computation in  $D = 4$ , in Section 3.

In  $D = 3$ , we find a cosmological constant proportional to the Euler number of the internal fourfold, which is the supersymmetric extension of the tadpole term  $\chi A_3$ .

Finally, in Section 5, we discuss type *IIB* compactifications to  $D = 4$  on Calabi-Yau. In particular, we extract a universal non-perturbative correction to the hypermultiplet metric, which exhibits  $SL(2, Z)$  invariance, and speculate about its possible extension to a quaternionic symmetry.

## 2. *M* Theory and type *IIA* theory on Calabi-Yau Threefolds

The bosonic action of the eleven-dimensional supergravity limit of *M* theory is given by

$$I_{11} = \frac{1}{2} \int_{M^{11}} d^{11}x \left[ \sqrt{-g} R - \frac{1}{2} F_4 \wedge *F_4 - \frac{1}{6} A_3 \wedge F_4 \wedge F_4 \right]. \quad (2.1)$$

This action should be implemented by a term predicted by membrane/fivebrane duality [3]. Indeed, cancelling the anomaly on the fivebrane worldvolume by a bulk contribution determines a coupling between the three-form potential and an eight-form polynomial in curvature.

$$I_{11}^{Lorentz} = \int_{M^{11}} A_3 \wedge \frac{1}{(2\pi)^4} \left[ -\frac{1}{768} (\text{tr} R^2)^2 + \frac{1}{192} \text{tr} R^4 \right]. \quad (2.2)$$

The gravitational constant and the membrane and fivebrane tensions are set to one.

The reductions of the effective theory on Calabi-Yau manifolds have been extensively discussed during the last year. In particular, in five dimensions it is well known (see, *e.g.*, [9,10]) that in addition to  $h_{(1,1)}$  vectors and  $h_{(2,1)} + 1$  hypermultiplets, the theory has a geometrical coupling term [11]

$$I_5 = -\frac{1}{12} C_{\Lambda\Sigma\Delta} \int_{M^5} A_1^\Lambda \wedge F_2^\Sigma \wedge F_2^\Delta. \quad (2.3)$$

The  $U(1)$  fields are normalized so that they couple to integer charges. On the other hand, the reduction of (2.2) yields an interaction of the form

$$I_5^{Lorentz} \sim \int_{M^5} \alpha_\Lambda A_1^\Lambda \wedge \text{tr} R^2. \quad (2.4)$$

As discussed in [12], (2.4) can be viewed as a bulk term needed for cancelling the anomalies due to the wrapping of the fivebrane on the CY four-cycles. The  $\alpha_\Lambda$  define the topological couplings:

$$\alpha_\Lambda = \frac{1}{16(2\pi)^2} \int_{X_6} \omega_\Lambda \wedge \text{tr} R^2, \quad (2.5)$$

where  $\Lambda = 1, \dots, h_{(1,1)}$  and  $\omega_\Lambda$  is the corresponding  $(1,1)$  harmonic form.

In analogy with the lower-dimensional Green-Schwarz terms, we claim that in eleven dimensions not only (2.2) should be added to the action but also its “supersymmetric” partner (the reference to supersymmetry is rather indirect here since it will be argued not directly in the eleven-dimensional theory but after its compactification):

$$\hat{t}^{\mu_1 \dots \mu_8} \hat{t}_{\nu_1 \dots \nu_8} R_{\mu_1 \mu_2}^{\nu_1 \nu_2} \dots R_{\mu_7 \mu_8}^{\nu_7 \nu_8}, \quad (2.6)$$

where for any antisymmetric matrix  $M$ ,  $\hat{t}^{\mu_1 \dots \mu_8} M_{\mu_1 \mu_2} \dots M_{\mu_7 \mu_8} \equiv 24 \text{tr} M^4 - 6(\text{tr} M^2)^2 + 1/2 \epsilon^{\mu_1 \dots \mu_8} M_{\mu_1 \mu_2} \dots M_{\mu_7 \mu_8} \equiv t_8 M^4 + \frac{1}{2} \epsilon \cdot M^4$ .<sup>1</sup> Clearly, (2.6) has a parity-violating piece, containing, one  $\epsilon$  tensor, and this is exactly the piece contributing to (2.2). The condition of existence of nowhere-vanishing spinors on the background imposes further constraints [14] and the integral over (2.6) can be replaced by an integral over a parity-preserving combination,  $t_8 t_8 R^4 - \frac{1}{4} \epsilon \epsilon R^4$ , which also appears in the expression of the four-loop  $\sigma$ -model beta-function [15], up to the relative sign which we discuss below. This is what we call the  $R^4$  term in the eleven-dimensional effective theory. By simple scaling argument, it can be shown that this reduces to a one-loop  $R^4$  term in type *IIA* theory.

As mentioned before, we would like to argue that the compactifications of this term yield results in complete agreement with supersymmetry. Let us concentrate on the  $D = 5$   $N = 1$  case obtained by compactification on a Calabi-Yau manifold. As was pointed out in ref. [12], here (in the decompactification limit of  $D = 4$   $N = 2$ ) all the instanton corrections are suppressed and we see the leading term in the (vector-valued) holomorphic coupling (2.4). The reduction of (2.6) will give the supersymmetric completion of this coupling, but also we will see a new effect in the universal hypermultiplet. Before we perform the reduction, we notice that besides the low derivative  $R^2$  and  $R$  terms, the eleven-dimensional  $R^4$  coupling is going to give rise also to a similar higher-derivative term in  $D = 5$ , which we do not discuss here.

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<sup>1</sup> We follow the conventions of [13,1,2]. In particular, we take the normalization of the eleven-dimensional term with two  $t_8$  to be  $\frac{\pi^2}{9 \cdot 2^7} \int d^{11}x \sqrt{-g} t_8 t_8 R^4$ . Moreover  $\epsilon$  denotes the totally antisymmetric tensor in 8 dimensions with Lorentzian signature, throughout the paper.

Let us first write  $R^4$  in  $D = 11$  in a more convenient form for our purposes:

$$Y_{11} \equiv \frac{-\pi^2}{32} \left[ 4 \int R \wedge R \wedge R \wedge R \wedge e \wedge e \wedge e - \int \sqrt{-g} R \cdot R \cdot R \cdot R \right], \quad (2.7)$$

where  $R \cdot R \cdot R \cdot R = 6t_8(4trR^4 - (trR^2)^2) = 12(R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda})^2 + \dots$ . We have written explicitly only the term essential for the threefold compactifications. On  $M_{11} = M_5 \times X_6$  there are two contributions from (2.7). The first, moduli dependent, reproduces the form familiar from the one-loop string formula for the gravitational holomorphic coupling

$$\frac{-1}{8}(c_2 \cdot \vec{J})R \wedge R \wedge e, \quad (2.8)$$

where the internal part is

$$\int_{X_6} R \wedge R \wedge e \wedge e = \frac{1}{(2\pi)^2}(c_2 \cdot \vec{J}). \quad (2.9)$$

Equation (2.8) is in fact the supersymmetric partner of (2.4). Upon reduction to  $D = 4$ , these two terms form the coupling  $\alpha_\Lambda Z^\Lambda R^2$  where  $Z$ 's are the complex fields in  $N = 2$  vector multiplets. Note that (2.8) receives a contribution from the fully contracted combination of four  $R$ 's. This contribution is exactly equal to the one from the wedged product, and this turns out to be an extremely important consistency check. Referring to [1] for more detailed discussion of this point, we just mention that the relative minus sign is fixed in Type *IIA*, while in type *IIB* the sign is ambiguous. As will be shown in Section 6,  $N = 2$  supersymmetry requires the four-dimensional analogue of (2.8) to have a vanishing coefficient for type *IIB* compactifications which therefore fixes the sign ambiguity to be the opposite from that of type *IIA*.

In ten dimensions, type *IIA* has also a tree-level  $R^4$  term which goes to zero in the eleven-dimensional limit [2]. The relative sign between  $t_8 t_8$  and  $\epsilon \epsilon$  is the same at tree level in both type *IIA* and *IIB* theories, following the four-loop  $\sigma$ -model computation [15]. It follows that type *IIA* has a relative sign flipped between tree-level and one-loop terms, while type *IIB* does not, consistently with  $SL(2, Z)$  symmetry. Note that because of this relative sign between  $t_8 t_8$  and  $\epsilon \epsilon$  terms the reduction of the tree-level terms to four dimensions gives a vanishing contribution to  $R^2$  for both type *IIA* and *IIB* as required by  $N = 2$  supersymmetry.

It is instructive to recall the *K3* compactification and discuss the analogue of (2.7) in seven dimensions. In this case, (2.9) is replaced by a constant equal to the Euler number

of  $K_3$  and the  $R^2$  coupling is simply the supersymmetric extension of the Chern-Simons gravitational couplings in  $D = 7$  proportional to the first Pontryagin number discussed in [3]. Moreover, upon compactification to six dimensions, it corresponds to a one-loop correction in the type *IIA* theory compactified on  $K3$ . Note that due to the sign flip between the  $t_8 t_8$  and  $\epsilon\epsilon$  term for the tree-level type *IIA* and *IIB* and for the one loop for *IIB* there is no  $R^2$  term at the tree level for both *IIA* and *IIB* and at the one loop level for *IIB*. The results for type *IIA* on  $K3$  are consistent with what one expects from the 6-dimensional heterotic-type *IIA* duality. Indeed it is easy to see that the type *IIA* tree-level and one-loop contributions to  $R^2$  are mapped via duality to heterotic one-loop and tree-level contributions respectively and in the heterotic theory compactified on  $T^4$  there is a tree-level  $R^2$  term but no one-loop term.

The second term which arises after integration on CY is moduli-independent and is proportional to the Euler number of the internal manifold:

$$\int_{X_6} R \wedge R \wedge R = \frac{1}{3! (2\pi)^3} \chi, \quad (2.10)$$

yielding to a correction to the Einstein term of the form  $\sqrt{-g} R \chi$ . This is not the whole story yet. There is a modification of the eleven-dimensional Einstein equation due to the higher curvature term. On  $M^{11} = M^5 \times X_6$ , the non-vanishing component is [16,17]

$$R_{i\bar{j}} = \partial_i \partial_{\bar{j}} X, \quad (2.11)$$

where  $X$  is the six-dimensional Euler integrand. This leads to a correction in the kinetic terms of  $h_{(1,1)} - 1$  vector moduli.

The overall effect is a modification of the usual five-dimensional action by a shift in the universal hypermultiplet<sup>2</sup> which contains the CY volume of  $X_6$ ,  $\mathcal{V} = e^{-2\phi_5}$  [9]. In fact, the scalar kinetic terms (with the properly redefined special coordinates  $t^\Lambda$ ) in five dimensions in the  $M$  theory frame should be obtained by reducing  $M$  theory on the Calabi-Yau space. The string calculation for type *IIA* presented in the next section implies that these kinetic terms are of the form:

$$\sqrt{-g} \left[ (e^{-2\phi_5} - \frac{1}{12\pi} \chi) (R + G_{\Lambda\Delta} \partial t^\Lambda \partial t^\Delta) + (e^{-2\phi_5} + \frac{1}{12\pi} \chi) G_{q\bar{q}} \partial q \partial \bar{q} \right], \quad (2.12)$$

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<sup>2</sup> Our discussion throughout this paper is applied only to the non-universal hypermultiplets, since we treat the universal one as constant.

Here  $q$  are the hypermultiplets and  $t^\Lambda$  are the 5-dimensional special coordinates subject to the constraint  $\frac{1}{6}t^\Lambda t^\Delta t^\Sigma C_{\Lambda\Delta\Sigma} = 1$ . The above constraint defines a  $h_{(1,1)} - 1$  dimensional surface [11,9], and the kinetic term for  $t^\Lambda$  contains the induced metric on this surface. A redefinition of the five-dimensional dilaton  $\phi_5$  is translated into a one-loop correction to the hypermultiplet metric  $G_{q\bar{q}} \rightarrow G_{q\bar{q}}[1 + e^{2\phi_5}\chi/(6\pi)]$ . This one-loop correction in the hypermultiplet geometry will be confirmed in the next section, where we show that this is indeed the case by performing a direct string computation in four dimensions.

In fact going to  $D = 4$ ,  $M$  theory compactification on  $CY \times S^1$  is believed to describe the strong coupling regime of type  $IIA$  on the same  $CY$ . In ten dimensions type  $IIA$  theory has  $R^4$  correction both at the tree level and at one loop level. As mentioned earlier the relative sign between the  $t_8 \cdot t_8$  term and  $\epsilon \cdot \epsilon$  term is opposite between the tree level and the one loop level. To be explicit, let us denote by  $Y_0$  and  $Y_2$  respectively the  $t_8 \cdot t_8$  and  $\epsilon \cdot \epsilon$  parts of  $R^4$  terms. Then the  $R^4$  term in the ten dimensional action for type  $IIA$  is obtained from the combination  $(t_8 + \frac{i}{2}\epsilon) \cdot (t_8 - \frac{i}{2}\epsilon)$  for the tree level while from  $(t_8 \cdot t_8 - \frac{1}{4}\epsilon \cdot \epsilon)$  for one loop, namely:

$$\int d^{10}x \sqrt{-g} [\zeta(3) e^{-2\phi} (Y_0 + Y_2) + (Y_0 - Y_2)] . \quad (2.13)$$

In type  $IIB$ , as we shall discuss in Section 5, the tree level and the one-loop terms will be identical. Equation of motion for the internal metric and the dilaton to this order gets contribution only from  $Y_0$  as discussed in [16,17]. The corrected metric turns out to be still Kähler though not Ricci flat. Denoting by  $\delta g$  the correction to the Calabi-Yau metric one finds

$$R_{i\bar{j}} = -\frac{1}{2} \partial_i \partial_{\bar{j}} Tr \delta g = \frac{1}{3!(2\pi)^3} (2\zeta(3) + e^{2\phi_0} \frac{2\pi^2}{3}) \partial_i \partial_{\bar{j}} X \quad (2.14)$$

$$\phi = \phi_0 + \frac{1}{12(2\pi)^3} (2\zeta(3) + e^{2\phi_0} \frac{2\pi^2}{3}) X \quad (2.15)$$

where  $\phi_0$  is the constant uncorrected dilaton.

The kinetic terms for scalars corresponding to the deformation of Kähler class and complex structures receive correction only from  $Y_0$  term, as  $Y_2$  will necessarily involve four 4-dimensional indices and hence four derivatives. It follows that the correction to the moduli metric is of the same form for tree level and one loop. Although we have not obtained the correction to the moduli metric directly by reducing the ten dimensional

action on the corrected Calabi-Yau space, the string calculation presented in the next section implies the following correction to the kinetic terms (in the string frame):

$$- \int d^4x \sqrt{-g} \frac{\chi}{(2\pi)^3} \left( e^{-2\phi_4} \frac{2\zeta(3)}{\mathcal{V}} + \frac{2\pi^2}{3} \right) (G_{\Lambda\Delta} \partial Z^\Lambda \partial \bar{Z}^\Delta - G_{q\bar{q}} \partial q \partial \bar{q}) \quad (2.16)$$

where  $q$  are the scalars of hypermultiplets (*i.e.*, complex structure moduli) and  $Z$  are the scalars in the vector multiplets (*i.e.*, complex Kähler moduli) orthogonal to the volume. The field  $\phi_4$  is the four dimensional dilaton and is related to the ten dimensional dilaton  $\phi$  appearing in eq.(2.15) in a somewhat complicated way owing to the equations (2.15) and (2.14). The ten dimensional dilaton kinetic term when expressed in terms of the 4-dimensional dilaton gives extra contribution to the kinetic term for the scalar field corresponding to the volume. It is for this reason that we have given the kinetic terms in eq.(2.16) only for scalars that are orthogonal to the volume. The kinetic term for the volume, which is a vector modulus, can be obtained by using the special geometry in terms of the prepotential given below. The metric  $G$  above refers to the original moduli metric for compactification on the uncorrected Calabi-Yau space. The relative sign between the Kähler and complex structure moduli above is consistent with the fact that under mirror symmetry  $\chi \rightarrow -\chi$  and the Kähler moduli get exchanged with the complex structure moduli.

The correction to the four dimensional Einstein term however comes entirely from  $Y_2$ , and therefore for *IIA* it is of opposite sign for the tree level and the one loop:

$$\int d^4x \sqrt{-g} \frac{\chi}{(2\pi)^3} \left( e^{-2\phi_4} \frac{2\zeta(3)}{\mathcal{V}} - \frac{2\pi^2}{3} \right) R \quad (2.17)$$

Going now to the Einstein frame one finds the following correction to the metric of Kähler moduli (orthogonal to the volume as described above) and of the complex structure moduli:

$$G_{\Lambda\Delta} \rightarrow \left( 1 - 4 \frac{\chi}{(2\pi)^3} \frac{\zeta(3)}{\mathcal{V}} \right) G_{\Lambda\Delta} \quad (2.18)$$

$$G_{q\bar{q}} \rightarrow \left( 1 + e^{2\phi_4} \frac{1}{6\pi} \chi \right) G_{q\bar{q}} \quad (2.19)$$

Noting that the Calabi-Yau volume  $\mathcal{V}$  is part of a vector multiplet and the four dimensional dilaton  $\phi_4$  is part of a hypermultiplet, we see that the above corrections are consistent with  $N = 2$  supersymmetry which dictates a decoupling of vector and hypermultiplet moduli. The correction to the vector multiplet geometry appears only at the tree level and in fact



corresponds exactly to the four loop correction to the prepotential<sup>3</sup> in *IIA* theory within the CY  $\sigma$ -model computation [18,15]

$$\mathcal{F}(Z) = \frac{1}{6} Z^\Lambda Z^\Delta Z^\Sigma C_{\Lambda\Delta\Sigma} - i \frac{\zeta(3)\chi}{2(2\pi)^3} + \dots \quad (2.20)$$

On the other hand the correction to the hypermultiplet geometry appears at one-loop level and is universal. We note that both tree-level and one-loop corrections are absent for CY threefolds with  $\chi = 0$ . This is essential for having a quantum-exact moduli space and second-quantized mirror symmetry in the (11, 11) CY threefold of [19]. The type *IIB* case will be analysed in Section 5.

We finish this section with a comment about fourfolds. The above-mentioned condition for existence of nowhere-vanishing spinors [14], gives rise to tadpole terms in  $D = 3$   $N = 2$  proportional to the Euler number of the internal manifold [20] that may be cancelled by appropriate membrane configuration [21]. It is easy to see that in addition to this tadpole, (2.7) gives rise to its supersymmetric extension  $\chi \cdot e \wedge e \wedge e$  which is simply a three-dimensional cosmological constant.

### 3. One-loop Correction to the Hypermultiplet Geometry

In this section we will obtain the 1-loop correction to the Einstein term as well as the metric for hypermultiplets directly from string theory. First let us consider the correction to the Einstein term. This has been earlier computed in [22] for the case of orbifold compactification, but here we shall do it for the general Calabi-Yau case. We will obtain the correction to the Einstein term by computing a 3-point function involving gravitons. From the effective field theory, this would be the sum of the irreducible 1-loop 3-graviton vertex plus the diagram corresponding to an intermediate graviton connecting the 3-graviton vertex at the tree level and the 1-loop 2-graviton vertex. Each of these diagrams is proportional to the 1-loop correction to the Einstein term, and it is easy to see that the proportionality constant is not zero.

The graviton vertex with the polarization  $h_{\mu\nu}$  in the zero ghost picture is

$$V_h(p) = h_{\mu\nu} : (\partial X^\mu + ip \cdot \psi \psi^\mu) (\bar{\partial} X^\nu + ip \cdot \tilde{\psi} \tilde{\psi}^\nu) e^{ip \cdot X} : , \quad (3.1)$$

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<sup>3</sup> In fact the prepotential (2.20) reproduces (2.18) in the large volume limit for the components of the metric orthogonal to  $\mathcal{V}$ . Furthermore, consistency of these equations requires an appropriate normalization of the volume.

where  $X^\mu$  and  $\psi^\mu$  and  $\tilde{\psi}^\mu$  are the bosonic and the left and right moving fermionic space-time coordinates (in the  $NSR$  formalism) and  $p$  refers to the 4-dimensional momentum. The contribution to the Einstein term being CP-even can come only from spin structures that correspond to left and the right sectors being both even or odd. Let us first consider the odd-odd spin structure. In this case, due to the presence of a holomorphic (anti-holomorphic) killing spinor, one of the graviton vertex must appear in  $(-1,-1)$  ghost picture:  $e^{-\phi-\tilde{\phi}}\psi^\mu\tilde{\psi}^\nu e^{ip\cdot X}$  where  $\phi$  and  $\tilde{\phi}$  are the bosonization of the left and the right moving superghosts. Moreover, the presence of the world-sheet gravitino zero modes imply the insertion of a left and right moving picture changing operator:  $P_L = e^\phi T_F$  (and similarly the right moving  $P_R$ ) where  $T_F = \psi^\mu \partial X^\mu + \dots$  is the superpartner of the world sheet stress energy tensor. In the odd-odd spin structure, there are four space-time fermion zero modes each in the left and the right sectors, and therefore one of the graviton vertex in  $(0,0)$  ghost picture must give the momentum dependent fermion bilinear piece. The four space-time left moving (and right moving) zero modes are soaked by the two fermions from this  $(0,0)$  ghost-picture graviton and one each from the  $(-1,-1)$  ghost-picture graviton and the  $T_F$  in the picture changing operator. The remaining bosonic part  $\partial X$  ( $\bar{\partial} X$ ) on the left (right) sector in the  $T_F$  and the remaining  $(0,0)$  ghost picture graviton, will contract with the corresponding right (left) moving parts through the propagator

$$\langle \partial X^\mu(z) \bar{\partial} X^\nu(w) \rangle = -\frac{\pi \delta_{\mu\nu}}{\text{Im}\tau} , \quad (3.2)$$

where  $\tau$  is the Teichmüller parameter of the world-sheet torus. After soaking the space-time fermion zero modes, the non-zero mode determinants of the space-time bosons and fermions cancel. In the internal  $N = 2$  conformal field theory describing the Calabi-Yau space, the amplitude is proportional to the Witten index  $\text{tr}(-1)^{F_L+F_R} q^{L_0} \bar{q}^{\bar{L}_0}$ , where  $F_L$  and  $F_R$  are the left and right moving charges with respect to the  $U(1)$ 's of the respective  $N = 2$  superconformal algebras,  $L_0$  and  $\bar{L}_0$  the left and right moving dimensions, and  $q = e^{2i\pi\tau}$ . The Witten Index for the Calabi-Yau space is just its Euler characteristic  $\chi$ . This therefore gives an amplitude which is quadratic in momenta with a coefficient proportional to  $\chi$ ,  $\int \frac{d^2\tau}{(\text{Im}\tau)^2} = 2\pi\chi/12$ . The dependence on  $\text{Im}\tau$  above follows from the fact that in four space-time dimensions the partition function comes with  $\text{Im}\tau^{-3}$  on the torus, while the two bosonic correlation functions  $\langle \partial X \bar{\partial} X \rangle$  give  $\text{Im}\tau^{-2}$  and the integrations over the positions of the three graviton vertices give  $\text{Im}\tau^3$ .

Let us now consider the contribution of the even-even spin structures to the 3-graviton amplitude involving two powers of momenta.. In the even-even spin structure we can use

the (0,0) ghost pictures for all the three gravitons with no insertion of picture changing operators. It is easy to see that one needs at least one fermion propagator from each of the left and the right moving sectors, as otherwise the sum over even spin structures would yield a vanishing result due to  $N = 2$  space-time supersymmetry. Thus from the left (and right) sector, two of the graviton vertices (say at  $z_1$  and  $z_2$ ) should provide the fermion bilinear pieces. The  $\partial X$  part of the third graviton vertex (say at  $z_3$ ) must contract with  $\bar{\partial} X$  part of another graviton vertex (say at  $z_1$ ). Thus the fermion bilinear pieces from the right moving sectors are provided by the gravitons at  $z_2$  and  $z_3$ . Therefore, the amplitude to begin with is of order  $p^4$ . The only way this can contribute to Einstein term, *i.e.*, quadratic in momenta, is if there is a contact term due to singularity in the integral over the positions of the graviton vertices. However, the correlation function of the fermions in the left sector is

$$S_e(z_1, z_2)^2 = -\partial_{z_1}^2 \log \theta_1(z_1 - z_2) + 2\pi i \partial_\tau \ln\left(\frac{\theta_e}{\eta}\right), \quad (3.3)$$

where  $S_e$  is the Szego kernel in the even spin structure labelled by  $e$ ,  $\theta_1$  and  $\theta_e$  are the odd and even Jacobi theta functions, and  $\eta$  is the Dedekind eta function. The right moving part gives a similar correlation function depending on  $(\bar{z}_2 - \bar{z}_3)$ . This shows that there is no singularity of the form  $|z_i - z_j|^2$ , and hence there is no contact term. It follows that the even-even spin structure can only contribute to the  $R^2$  term and not to the Einstein term. This was indeed expected from the 1-loop 10-dimensional  $R^4$  term which contains two pieces: one appearing with the combination  $t_8 \cdot t_8$  and the other with  $\epsilon \cdot \epsilon$ , that come respectively in the even-even and odd-odd spin structures. The first term above does not contribute to the Einstein term while the second one gives a contribution proportional to the Euler characteristic of the Calabi-Yau 3-fold.

Now let us turn to the question of the 1-loop correction to the metric of the vector- and hyper-multiplet moduli. How do we extract this correction from an on-shell amplitude. The first thing to note is that the 2-point function of the corresponding scalars is zero on-shell. Moreover the 3-point function involving two scalars and a graviton at one-loop level gets contribution from the irreducible 1-loop 3-point vertex as well as the reducible diagram involving the wave function renormalization of the scalar with the tree-level 3-point vertex of two scalars and a graviton. Each of the above contributions is proportional to the one loop correction to the metric and in fact they cancel each other. There is also another reducible diagram involving the tree-level 3-point vertex of two scalars and a

graviton together with the one-loop graviton self-energy but this depends only on the one loop correction to  $R$  term and does not carry the information of the 1-loop correction to the metric. Thus, in order to get the information about the one-loop correction to the metric, we must compute a 1-loop 4-point amplitude involving two scalars and two gravitons. Note that the corresponding string amplitude will also include one-loop correction to the Einstein term via one-loop 2 or 3-graviton vertex with the tree-level kinetic term of the scalars. In fact the sum of all these diagrams gives the correct one-loop correction to the metric in the one-loop corrected Einstein frame. Thus, a vanishing (non-vanishing) string amplitude would imply vanishing (non-vanishing) one-loop correction to the moduli metric in the Einstein frame, in which the coefficient of  $R$ , including the one-loop correction, has been scaled to unity.

We are thus led to computing a 4-point amplitude  $\langle hhq\bar{q} \rangle$  where  $h$  are graviton vertices and  $q$  and  $\bar{q}$  are the  $NS$ - $NS$  scalar and its complex conjugate respectively. The vertex operator for the scalar  $q$  is

$$V_q(p) =: P_L P_R e^{-\phi - \tilde{\phi}} \Psi e^{ip \cdot X} : , \quad (3.4)$$

where  $P_L$  and  $P_R$  are the picture changing operators and  $\Psi$  is a chiral-chiral or chiral-antichiral operator for type  $IIA$  hypermultiplet or vector multiplet, respectively, and vice versa for type  $IIB$ . The vertex operator for  $\bar{q}$  is just the complex conjugate of the above.

The amplitude in question is CP-even and therefore gets contribution only from even-even and odd-odd sectors. Let us first consider the even-even sector. As mentioned earlier, in order to get a non-vanishing result after the spin-structure sum the amplitude must involve fermion correlators from both the left and the right sectors. The lowest possible power of momenta is therefore  $p^4$ , with  $p^2$  coming from each of the two sectors. In order to obtain one-loop correction to the metric, which is quadratic in momenta, we must look for possible singularities in the integrals over the positions of the vertices which could give a  $1/p^2$  pole. Now  $p^4$  can appear in three ways:

a) two gravitons provide  $p^2$  from left as well as right sector. In this case the correlation function is proportional to  $|S_e(z-w)|^4$ ,  $z$  and  $w$  being the positions of the graviton vertices, which after integration using the formula (3.3) yields a constant. The scalar operators  $\Phi = \oint T_F \oint \tilde{T}_F \Psi$  and its complex conjugate give rise to the second derivative  $\partial_q \partial_{\bar{q}}$  of the partition function of the internal conformal field theory in the even-even spin structure and therefore there is no singularity in the position integrals of the  $q$  and  $\bar{q}$  vertices. Thus, there is no contact term of the form  $1/p^2$ .

b) two gravitons provide  $p^2$  from the left sector and the two scalars provide  $p^2$  from the right. In this case the right moving parts of the graviton vertices  $\bar{\partial}X$  necessarily bring down more powers of momenta.

c) the two scalar vertices provide  $p^2$  from both the sectors and the left moving bosonic parts  $\partial X$  of the graviton vertices contract with the corresponding right moving parts as in the computation of the one-loop correction to  $R$  above. The correlation function now is  $S_e(z-w)\bar{S}_{e'}(\bar{z}-\bar{w})\langle\Psi(z,\bar{z})\bar{\Psi}(w,\bar{w})\rangle_{e,e'}$ , where  $e$  and  $e'$  are the even spin-structures on the left and the right sectors correspondingly, and  $z$  and  $w$  are the positions of the vertex operators for  $q$  and  $\bar{q}$ , respectively. This correlation function has the leading singularity structure  $1/|z-w|^4$  which is proportional to the tree-level metric  $G_{q\bar{q}}^{(0)}$ . Now the spin structure dependence in the internal conformal field theory describing the Calabi-Yau space, enters only through the charge lattice of the  $U(1)$ 's of the left and right moving  $N=2$  superconformal algebras (see for example [23]). Thus, we can extract the spin structure dependent part of the correlation function for the left moving sector with the result:

$$\theta_e(z-w)Z_e(\pm\frac{1}{\sqrt{3}}(z-w)) , \quad (3.5)$$

where  $Z_e$  is the  $N=2$   $U(1)$  charge lattice in the same spin-structure and the  $\pm$  sign in the argument of  $Z_e$  above is for  $q$  being chiral or anti-chiral in the left moving sector, following the corresponding  $U(1)$  charges  $\pm\frac{1}{\sqrt{3}}$ .

One can now carry out the spin structure sum using the formula:

$$\sum_e \theta_e(a)Z_e(b) + \theta_1(a)Z_1(b) = 2\theta_1(\frac{a+\sqrt{3}b}{2})Z_1(\frac{\sqrt{3}a-b}{2}) , \quad (3.6)$$

where  $Z_1$  is the  $U(1)$  charge lattice in the odd spin structure. Since  $\theta_1(z-w)$  goes to zero as  $z \rightarrow w$ , the leading singularity in the correlation function, including the right moving sector is  $1/|z-w|^2$  which is precisely the singularity needed to produce a  $1/p^2$  pole. This means that in the above equation we can set the argument of  $Z_1$  equal to zero. It then follows from the eq.(3.6) that the contribution of the sum over the even spin structures to the metric correction is equal to plus or minus  $\theta'_1(0)Z_1(0)$  for  $q$  being chiral or anti-chiral, respectively. Note that  $\theta'_1(0)$  exactly cancels the contribution of the two space-time bosonic non-zero mode determinants.

In the odd-odd spin structure, as discussed earlier we have to use one of the vertex operators (say  $q$ ) in  $(-1,-1)$  ghost picture and we have to insert also a picture changing

operator each for the left and right moving sector. After soaking the space-time fermion zero modes, and contracting the  $\partial X$  in one of the graviton vertex with the right moving part via the correlator (3.2), we find again a  $p^4$  term. The space-time bosonic and fermionic non-zero mode determinants cancel as usual. In the internal theory on the other hand there is a correlation function  $\langle \Psi(z, \bar{z}) \bar{\Psi}(w, \bar{w}) \rangle$  which has a leading singularity  $1/|z - w|^2$  with coefficient  $G_{q\bar{q}}^{(0)} |Z_1(0)|^2$ , which upon integration gives rise to a  $1/p^2$  contact term. This result is therefore exactly the one obtained for the sum over the even spin structures.

Combining the above contributions and taking into account the relative sign between the even-even and odd-odd spin structures for type *IIA* and *IIB*, we find that the total result vanishes if  $q$  is chiral-antichiral for *IIA* or chiral-chiral for *IIB*. This means that in both type *II* theories, the one-loop correction to the metric of vector multiplets vanishes (in the one-loop corrected Einstein frame). Of course, this is expected from the non-renormalization of the vector moduli space due to the fact that the dilaton in type *II* theories on Calabi-Yau spaces sits in hypermultiplet.

On the other hand for hypermultiplets (*i.e.*,  $q$  being chiral-chiral for *IIA* or chiral-antichiral for *IIB*) the even-even sector adds up to the odd-odd sector and yields a total of twice the latter. The result as we have seen above, after extracting the leading singularity, is just the partition function of the internal conformal field theory in the odd-odd spin structure which is proportional to the Euler characteristic  $\chi$  of the Calabi-Yau space. To summarize, one-loop string computation shows that while the metric for vector multiplets is not renormalized (as expected), the metric of hypermultiplets gets a universal one-loop correction proportional to the Euler characteristic of the Calabi-Yau space times the tree-level metric.

Finally we would like to comment on the implication of the string computation presented in this section to the kinetic terms in eq.(2.16). First thing to note is that in the odd-odd spin structure the contribution comes entirely from the leading singularity in the  $\Psi$ - $\bar{\Psi}$  OPE. This corresponds to a factorized diagram involving a tree level two scalar and a graviton vertex with the one loop three graviton vertex, the first being proportional to the tree level metric of the scalars while the second one represents the one loop correction to the Einstein term  $R$ . Indeed as we had mentioned earlier the correction to  $R$  comes from the  $\epsilon \cdot \epsilon$  part of  $R^4$  term in 10-dimensions which appears in the odd-odd spin structure. The even-even spin structure on the other hand gives the  $t_8 \cdot t_8$  part of  $R^4$  term and hence does not give correction to the Einstein term. Therefore the result appearing in the even-even spin structure should correspond to the kinetic terms (in the string frame)

given in eq.(2.16). Indeed this is consistent with the fact that in the string calculation the leading singularity between the vertex operators for the two scalars cancels upon summing over the even-even spin structures. What survives is in fact the subleading singularity which should be interpreted as the irreducible part of the correction to the kinetic terms. Furthermore the even-even spin structure contribute to the Kähler and complex structure moduli equal but with opposite sign which is consistent with eq.(2.16).

#### 4. Chiral Fivebrane in Type *IIB* Theory

In this section, we show the emergence of a chiral fivebrane with a (2,0) tensor multiplet supported on its worldvolume in type *IIB* theory compactified on  $S^1$ . This can be predicted on the basis of duality of this theory with type *IIA* on  $S^1$  by matching the corresponding branes. The fact that counting matches is well-known [4], but the fact that type *IIA* theory contains a chiral fivebrane in ten dimensions while type *IIB* does not, suggests that the fivebrane which is a magnetic source for the vector  $A_\mu = g_{\mu 9}$  in type *IIB* must be chiral. The existence of this third fivebrane is also in agreement with the *IIB* supersymmetry algebra in ten dimensions which has a triplet of self-dual fivebrane central charges [24]<sup>4</sup>.

A direct zero mode analysis can be done following [26], but here we present only a simple argument based on worldvolume anomalies. In eleven dimensions, it was pointed out [27,28] that, in the presence of a fivebrane, a term representing the coupling of an anti-self-dual three-form field strength  $T_3$  on the fivebrane worldvolume is necessary to cancel the anomaly from the interaction  $\int_{M^{11}} A_3 \wedge F_4 \wedge F_4$ . This can be seen as follows. In the presence of a fivebrane with charge  $m$ ,

$$dF_4 = m\delta_V, \tag{4.1}$$

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<sup>4</sup> A similar mechanism can be seen when deriving the five-dimensional  $N = 8$  supersymmetry algebra from eleven dimensional superalgebra with two- and fivebrane central charges. In addition to 27 (scalar) electric and 27 (vector) magnetic charges, an  $E_6$  singlet emerges which has the interpretation of the KK monopole charge [25]. When the theory is reduced to four dimensions, these charges together with the extra KK charge complete the 56-dimensional representation of  $E_7$ .

where  $\delta_V$  is supported on the fivebrane worldvolume  $V$  (*i.e.* it integrates to 1 on the space transverse to the fivebrane). So, under  $\delta A_3 = d\Lambda_2$ ,

$$\begin{aligned} \frac{1}{12} \delta \left( \int_{M^{11}} A_3 \wedge F_4 \wedge F_4 \right) &= \frac{1}{4} \int_{M^{11}} d\Lambda_2 \wedge F_4 \wedge F_4 \\ &= -\frac{m}{2} \int_V \Lambda_2 \wedge F_4. \end{aligned} \quad (4.2)$$

This anomaly needs to be cancelled by a term

$$\frac{m}{2} \int_V T_3 \wedge A_3, \quad (4.3)$$

where  $T_3$  is the anti-self-dual three-form field strength on the fivebrane worldvolume and  $dT_3 = F_4$ .

Similarly, the kinetic term for the self-dual four-form field in type *IIB* yields in nine dimensions, besides the kinetic term of the three-form field, a Chern-Simons (CS) coupling involving the vector  $A_\mu$ :  $\int_{M^9} A_1 \wedge F_4 \wedge F_4$ . Here, we don't consider other couplings, involving  $A_3$  with Neveu Schwarz (*NS*) and Ramond-Ramond (*RR*) two-form fields inherited from ten dimensions since they are not relevant for our analysis. The fivebrane source with charge  $m$  is now,

$$dF_2 = m\delta_V, \quad (4.4)$$

and under  $\delta A_3 = d\Lambda_2$ , the CS coupling develops exactly the same anomaly as before which can be remedied only by introducing a coupling to a tensor on the worldvolume (4.3). Again, this is an indirect argument which can be confirmed by the zero-mode analysis. Similar analysis in type *IIA* shows that the fivebrane charged under  $A_\mu = g_{\mu 9}$  is non-chiral. Thus in nine dimensions, the quantum consistency of the theory requires a coupling involving an eight-form in curvature that is formally given by the gravitational anomaly of a six-dimensional  $(2, 0)$  tensor multiplet:

$$\int A_1 \wedge I_8(R). \quad (4.5)$$

Note that under the  $U$ -duality group  $SL(2, Z)$  the three nine-dimensional vectors form a doublet (non-chiral fivebrane) and a singlet (chiral fivebrane). While the singlet in type *IIB* is the  $A_\mu = g_{\mu 9}$  vector, in type *IIA* it is the  $A_\mu = B_{\mu 9}$  which is consistent with the fact that the  $T$ -duality connecting the two theories in nine dimensions interchanges the internal components of the metric and antisymmetric tensor. Thus we have shown the existence of a supersymmetric partner of the  $R^4$  terms in nine dimensions.



A similar mechanism arises in the reduction of six-dimensional chiral theories to five dimensions. A theory with  $n_T$  tensor multiplets and  $n_V$  vector multiplets have gravitational Chern-Simons couplings (only the scalars belonging to the tensor multiplets couple to  $R^2$  terms). However, the five-dimensional theory contains  $n_T + n_V + 1$  vectors and, generally speaking, all of them have gravitational couplings. In particular, we observe again that the extra vector multiplet coming from the metric acquires a gravitational coupling due to the fact that the BPS string, which is magnetically charged under this multiplet, is chiral.

## 5. Universal Corrections to $IIB$ Theory

In this section, we discuss modifications of the two-derivative terms in the effective action of type  $IIB$  compactifications on CY threefolds to four dimensions. In ten dimensions, the Einstein term together with the  $R^4$  action (2.7) can be written, in the string frame, as [1]:

$$\sim \int_{M_{10}} \sqrt{-g} [e^{-2\phi_{10}} R + e^{-\phi_{10}} f(\rho, \bar{\rho}) Y^{IIB}] , \quad (5.1)$$

where  $\rho = \rho_1 + i\rho_2$  with  $\rho_1$  the  $RR$  scalar and  $\rho_2 = e^{-\phi_{10}}$ . The function  $Y^{IIB} = \frac{1}{3 \cdot 2^8} (t_8 \cdot t_8 R^4 + \frac{1}{4} \epsilon \cdot \epsilon R^4) \equiv Y_0 + Y_2$  (in the notation of Section 2), where we have used the relative sign that appears at the tree level of  $IIA$  theory. Indeed at the tree level there is no difference in  $R^4$  term for  $IIA$  and  $IIB$  theory. The  $SL(2, Z)$  invariance of  $IIB$  then fixes the sign of  $R^4$  terms for all perturbative and non-perturbative corrections. This of course implies that one-loop  $\epsilon \cdot \epsilon$  terms for  $IIA$  and  $IIB$  have opposite signs. We shall also see below that the absence of hypermultiplet dependence of the four dimensional  $R^2$  term (which is dictated by supersymmetry) implies precisely this choice of relative sign. The function  $f$  is  $SL(2, Z)$  modular invariant and its explicit form has been conjectured in ref. [1] :

$$f(\rho, \bar{\rho}) = \sum'_{n,m \in Z} \frac{\rho_2^{3/2}}{|n + m\rho|^3} = 2\zeta(3)\rho_2^{3/2} + \frac{2\pi^2}{3}\rho_2^{-1/2} + \dots \quad (5.2)$$

The first two terms in the r.h.s. of (5.2) correspond to the tree-level and one-loop contributions while the dots stand for the instanton sum.

Upon compactification to four dimensions and using (2.10), one finds

$$\int_{M_4} \left[ e^{-2\phi_4} + \frac{\chi}{(2\pi)^3} (2\zeta(3) \frac{e^{-2\phi_4}}{\mathcal{V}} + \frac{2\pi^2}{3} + \dots) \right] \sqrt{-g} R, \quad (5.3)$$

where  $\phi_4$  is the four-dimensional dilaton,  $e^{-2\phi_4} = e^{-2\phi_{10}}\mathcal{V}$  plus corrections owing to equations similar to (2.14) and (2.15). Here  $\mathcal{V}$  is the CY volume. Note that the relative sign between the tree level and the one-loop term above is opposite to that of type *IIA* (2.17). On the other hand since the moduli metric gets contribution only from the  $Y_0$  term above it should have the same form as in (2.16) for the *IIA* case:

$$\int d^4x \sqrt{-g} \frac{\chi}{(2\pi)^3} (e^{-2\phi_4} \frac{2\zeta(3)}{\mathcal{V}} + \frac{2\pi^2}{3} + \dots) (G_{\Lambda\Delta} \partial Z^\Lambda \partial \bar{Z}^\Delta - G_{q\bar{q}} \partial q \partial \bar{q}) \quad (5.4)$$

Here we have flipped the overall sign as compared to (2.16) due to the fact that in *IIB* Kähler and complex structure moduli correspond to hyper and vector multiplets as opposed to the *IIA* case where they correspond to vector and hypers respectively <sup>5</sup>. Just as in (2.16), the above formula is true only for Kähler moduli (which are hypers in the present case) that are orthogonal to the volume.

By redefining the dilaton as in (2.12) and going to the Einstein frame one finds the following corrections to the metric for the vector moduli and that for the hyper moduli that are orthogonal to the volume:

$$G_{\Lambda\Delta} \rightarrow G_{\Lambda\Delta} \quad (5.5)$$

$$G_{q\bar{q}} \rightarrow [1 - \frac{2\chi}{(2\pi)^3} (\frac{2\zeta(3)}{\mathcal{V}} + e^{2\phi_4} \frac{2\pi^2}{3} + \dots)] G_{q\bar{q}} \quad (5.6)$$

Thus the metric of the vector multiplet is not corrected even at tree level. On the other hand eq.(5.6) implies a universal correction to the hypermultiplet quaternionic geometry, in the large volume limit. In fact, the second (one-loop) term was already obtained in Section 3 by a direct string calculation. The tree-level term  $\sim \zeta(3)\chi$  is the contribution which should come from the classical  $c$ -map [29,30], due to the modification of the prepotential from the zero-instanton sector. In other words, this modification can be identified on the type *IIA* side as a four-loop contribution to the prepotential within the CY  $\sigma$ -model computation and was discussed in Section 2 (see (2.20)). Note that although the tree level correction implied in (5.6) is given here only for the hypers that are orthogonal to the volume, it can be extended to include volume by using the tree level special geometry for the NS-NS part of the non-universal hypers. From this argument we learn that type *IIB* hypermultiplet geometry, in the large volume limit, receives tree-level, one-loop and

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<sup>5</sup> Note that in terms of Kähler and complex structure moduli, (5.4) and (2.16) are identical. Throughout the paper, we denote the vector and hyper moduli as  $Z^\Lambda$  and  $q$  respectively.

non-perturbative corrections<sup>6</sup>. Again, such corrections are absent for CY's with  $\chi = 0$  [19].

One type of universal non-perturbative corrections to hypermultiplets has been discussed in [31] and involves fivebrane instantons. In type *IIB* theory with a non-vanishing  $RR$  fields, there can be two euclidean fivebranes wrapped around the CY space, and thus one expects that the results of [31] are modified. It is known that the two dilatons and two antisymmetric tensors form the universal hypermultiplet and their classical moduli space is locally  $\frac{SU(2,1)}{SU(2) \times U(1)}$ . The effect of the wrapped instantons should be combined together with the instanton corrections present already in ten dimensions, and as a result the whole system should respect the quaternionic structure. Indeed, this multiplet is not sensitive to the internal space, and thus its monodromy seems to be a direct generalization of  $SL(2, Z)$  and is of purely gravitational nature. It is suggestive that  $SL(2, Z)$  is enlarged to  $SU(2, 1, Z)$ , and leads us to a speculation that in four dimensions the  $SL(2, Z)$  modular function  $f$  conjectured in [1] should be promoted to a quaternion-valued function  $\hat{f}(Q)$ , where  $Q$  is built from the universal hypermultiplet<sup>7</sup>. Another argument in favour of  $SU(2, 1, Z)$  is that (at least on a manifold mirror to one with  $h_{(2,1)} = 0$  [32]) the quaternionic geometry is likely to have both ten-dimensional  $SL(2, Z)$   $U$ -duality as well as four-dimensional  $SL(2, Z)$   $S$ -duality, with a common  $Z_2$  symmetry that inverts the coupling constant but two different Peccei-Quinn symmetries (one  $RR$ , one  $NS$ - $NS$ ). In a sense, this is a complimentary case to the one studied in [33] where  $h_{(2,1)}$  was taken to be one and the universal multiplet was frozen. One should note though that in our case the hyper-Kähler limit is trivial.

## 6. Concluding Remarks

After having explained in Section 2 the Kähler class dependence of  $R^2$  terms for a generic CY compactification of type *IIA* theory as a consequence of  $R^4$  coupling, we address now the question of the origin of additional terms required by spacetime supersymmetry. Recall that in four dimensions  $R^2$  couplings come from a supersymmetric action of the type  $\mathcal{F}_1 W^2|_{\text{F-term}}$ , where  $W$  is the  $N = 2$  chiral Weyl superfield and  $\mathcal{F}_1$  is a function of chiral vector superfields [23]. Expanding in components, one finds that in addition to

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<sup>6</sup> Note that only the tree-level term is consistent with the classical  $c$ -map since it can be reabsorbed in a modification of the prepotential.

<sup>7</sup>  $\hat{f}(Q)$  should have the basic properties of  $f(\rho, \bar{\rho})$  which should be reproduced in an appropriate (decompactification) limit when the fivebrane instantons are suppressed.

$R^2$  there are other four-derivative terms of the form  $T^2 F^2$  and  $TFR$ , where  $T$  and  $F$  are the field-strengths of the  $N = 2$  graviphoton and “matter” vectors, respectively. The graviphoton originates from the eleven-dimensional metric while the gauge fields come from the three-form  $A_3$ . Since in the large volume limit, the function of the moduli  $f(t) = \alpha_\Lambda t^\Lambda$  coupled to  $R^2$  is linear, the  $T^2 F^2$  term drops out. Moreover, since  $\alpha_\Lambda$  are real, only the imaginary part of  $FTR$  term in the superfield expansion will contribute. This term comes from the original  $A_3 \wedge X_8(R)$  term. To see this let's integrate (2.4) by part to get  $\alpha_\Lambda F_2^\Lambda \wedge \omega_3$  and note that upon reduction on  $S^1$ , this contains

$$\alpha_\Lambda F_2^\Lambda \wedge RT, \quad (6.1)$$

where two-form  $RT$  stands for  $R_{\mu\nu}^{\sigma\lambda} T_{\sigma\lambda}$ .

Turning to type *IIB* theory, we first note as a consistency check that according to special geometry there should be no loop corrections associated with the Kähler class moduli. Indeed, with the different choice of relative sign in (2.7) the two contributions to the four-dimensional  $(c_2 \cdot \vec{J})R^2$  term cancel and there are no such corrections. Similar cancellation holds for the  $R^2$  couplings in  $D = 6$  when type *IIB* is compactified on  $K_3$ .

For the discussion of complex structure dependence of  $R^2$  term, we first turn to  $N = 4$  case. The reduction of type *IIA* on  $K3 \times T^2$  gives a term  $tR^2$ , where  $t$  is the Kähler modulus of  $T^2$ , and by duality we expect a similar term in type *IIB* where  $t$  modulus is replaced by the complex structure  $U$ . The answer is rather obvious if one first compactifies on  $S^1$ . Then the anomalous coupling associated with chiral fivebrane discussed in Section 4 will be further reduced to the term of the right form. Note that if one now takes for example a CY that has an orbifold limit  $K3 \times T^2/Z_2$ , the off-diagonal component of the metric on  $T^2$  (*i.e.* the complex structure  $U$ ) survives the orbifolding, and one again gets an  $R^2$  coupling depending on the complex structure.

*Note added:* The recent paper hep-th/9706195 by A. Strominger overlaps with some of our results, in particular concerning the one-loop correction to the universal hypermultiplet.

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